

NAG Toolbox for MATLAB

g08ae

1 Purpose

g08ae performs the Friedman two-way analysis of variance by ranks on k related samples of size n .

2 Syntax

```
[fr, p, ifail] = g08ae(x, k, 'n', n)
```

3 Description

The Friedman test investigates the score differences between k matched samples of size n , the scores in the i th sample being denoted by

$$x_{i1}, x_{i2}, \dots, x_{in}.$$

(Thus the sample scores may be regarded as a two-way table with k rows and n columns.) The hypothesis under test, H_0 , often called the null hypothesis, is that the samples come from the same population, and this is to be tested against the alternative hypothesis H_1 that they come from different populations.

The test is based on the observed distribution of score rankings between the matched observations in different samples.

The test proceeds as follows

- (a) The scores in each column are ranked, r_{ij} denoting the rank within column j of the observation in row i . Average ranks are assigned to tied scores.
- (b) The ranks are summed over each row to give rank sums $t_i = \sum_{j=1}^n r_{ij}$, for $i = 1, 2, \dots, k$.
- (c) The Friedman test statistic **fr** is computed, where

$$\mathbf{fr} = \frac{12}{nk(k+1)} \sum_{i=1}^k \left\{ t_i - \frac{1}{2}n(k+1) \right\}^2.$$

g08ae returns the value of **fr**, and also an approximation, p , to the significance of this value. (**fr** approximately follows a χ^2_{k-1} distribution, so large values of **fr** imply rejection of H_0). H_0 is rejected by a test of chosen size α if $p < \alpha$. The approximation p is acceptable unless $k = 4$ and $n < 5$, or $k = 3$ and $n < 10$, or $k = 2$ and $n < 20$; for $k = 3$ or 4 , tables should be consulted (e.g., **n** of Siegel 1956); for $k = 2$ the Sign test (see g08aa) or Wilcoxon test (see g08ag) is in any case more appropriate.

4 References

Siegel S 1956 *Non-parametric Statistics for the Behavioral Sciences* McGraw–Hill

5 Parameters

5.1 Compulsory Input Parameters

1: **x(ldx,n)** – double array

ldx, the first dimension of the array, must be at least **k**.

x(i,j) must be set to the value, x_{ij} , of observation j in sample i , for $i = 1, 2, \dots, k$; $j = 1, 2, \dots, n$.

- 2: **k – int32 scalar**
 k , the number of samples.
Constraint: $k > 1$.

5.2 Optional Input Parameters

- 1: **n – int32 scalar**
Default: The dimension of the array **x**.
 n , the size of each sample.
Constraint: $n \geq 1$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldx, w1, w2

5.4 Output Parameters

- 1: **fr – double scalar**
The value of the Friedman test statistic, **fr**.
- 2: **p – double scalar**
The approximate significance, p , of the Friedman test statistic.
- 3: **ifail – int32 scalar**
0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, $n < 1$.

ifail = 2

On entry, $ldx < k$.

ifail = 3

On entry, $k \leq 1$.

7 Accuracy

For estimates of the accuracy of the significance p , see g01ec. The χ^2 approximation is acceptable unless $k = 4$ and $n < 5$, or $k = 3$ and $n < 10$, or $k = 2$ and $n < 20$.

8 Further Comments

The time taken by g08ae is approximately proportional to the product nk .

If $k = 2$, the Sign test (see g08aa) or Wilcoxon test (see g08ag) is more appropriate.

9 Example

```
x = [1, 2, 1, 1, 3, 2, 3, 1, 3, 3, 2, 2, 3, 2, 2.5, 3, 3, 2;  
      3, 3, 3, 2, 1, 3, 2, 3, 1, 1, 3, 3, 2, 3, 2.5, 2, 2, 3;  
      2, 1, 2, 3, 2, 1, 1, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1];  
k = int32(3);  
[fr, p, ifail] = g08ae(x, k)  
  
fr =  
    8.5833  
p =  
    0.0137  
ifail =  
        0
```